



Enhancement of stimulated Brillouin scattering in metamaterials

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Abstract – We calculate the effective stimulated Brillouin scattering (SBS) gain coefficient for an all-dielectric cubic metamaterial. Using conventional effective medium procedures and perturbation methods, we demonstrate for the first time a complete procedure for calculating the effective acoustic and optoacoustic parameters present in the gain coefficient. The objective is to enhance SBS in technologically important materials for practical applications. An order of magnitude enhancement in the gain coefficient of silicon is achieved when introducing a cubic suspension of chalcogenide spheres.

I. INTRODUCTION

Stimulated Brillouin scattering (SBS) is a nonlinear scattering process which describes when an incident electromagnetic wave generates both a backward propagating electromagnetic wave and a coherent acoustic wave [1]. Two principal effects are observed with SBS; an acoustic wave is generated, and the backward propagating electromagnetic field is downshifted in frequency (as the acoustic wave acts as a traveling diffraction grating in the material). These effects arise due to the optoacoustic properties of the material; the electrostriction and photoelasticity, which describe when an electric field generates a mechanical strain field in the material, and when a strain field induces a change in the inverse permittivity of the medium, respectively [3, 7]. Materials with strong SBS properties are extremely useful for a wide range of practical applications, and have been successfully used to make sensors, photonic filters, and Brillouin lasers [2]. Accordingly, there is considerable interest in enhancing SBS in technologically important materials which exhibit poor SBS properties, such as silicon. We provide a complete procedure for determining the effective photoelasticity and effective acoustic losses in a dielectric metamaterial with cubic symmetry, and demonstrate that electrostriction is the dominant effect in the effective SBS gain coefficient of structured silicon.

II. SBS GAIN COEFFICIENT FOR CONVENTIONAL DIELECTRIC MATERIALS

In the absence of optical losses, the intrinsic SBS power gain coefficient is given by [1]

$$g_P(\Omega) = \frac{4\pi^2 \gamma_{\text{xxyy}}^2}{nc\lambda_1^2 \rho V_A \Gamma_B} \left(\frac{(\Gamma_B/2)^2}{(\Omega_B - \Omega)^2 + (\Gamma_B/2)^2} \right), \quad (1)$$

where $\gamma_{\text{xxyy}} = n^4 p_{\text{xxyy}}$ is an element of the electrostrictive stress tensor, p_{xxyy} is an element of the photoelastic tensor, Γ_B is the Brillouin line width (a measure of the acoustic loss), n is the refractive index, c is the speed of light in vacuum, λ_1 is the incident optical wavelength in vacuum, ρ is the material density, V_A is the long-wavelength longitudinal acoustic phase velocity, Ω is the angular frequency of the acoustic wave, and Ω_B denotes the Brillouin frequency shift (the velocity of the acoustic wave at the SBS resonance).

Although there is an extensive range of procedures for determining the effective optical and acoustic parameters in the expression above, there are no established methods for computing the $p_{\text{xxyy}}^{\text{eff}}$ and Γ_B^{eff} of a metamaterial. In the next section, we consider the $p_{\text{xxyy}}^{\text{eff}}$ in structures with cubic symmetry, beginning with formal definitions.



III. EFFECTIVE PHOTOELASTIC PARAMETER

The photoelastic tensor of a material is implicitly defined in Einstein notation through the identity [3]

$$\Delta(\varepsilon_{ij}^{-1}) = p_{ijkl} s_{kl}, \quad (2)$$

where ε_{ij} denotes the relative permittivity tensor and s_{kl} denotes the strain. We determine $p_{\text{xxxy}}^{\text{eff}}$ of our metamaterial by perturbing the boundaries of the unit cell to approximate a simple strain. By comparing the change in the effective inverse permittivity tensor under such a strain, and using the symmetry properties of cubic materials, the desired $p_{\text{xxxy}}^{\text{eff}}$ is recovered.

Assuming the principal axes of our cubic metamaterial are aligned with the Cartesian coordinate axes, the effective permittivity of our metamaterial is determined by equating the volume-averaged energy density to an effective energy density [4]

$$U_{\text{avg}} = \frac{1}{2} \varepsilon_0 \langle \varepsilon_{ij} |E_j|^2 \rangle = U_{\text{eff}} = \frac{1}{2} \varepsilon_0 \varepsilon_{ij}^{\text{eff}} |E_j|^2, \quad (3)$$

where $\langle \cdot \rangle$ denotes the volume average over the cell, ε_0 is the vacuum permittivity and E_j is the electric field distribution of the Bloch mode. An invertible linear system in $\varepsilon_{ij}^{\text{eff}}$ is then populated using a selection of optical Bloch modes with wave vectors near the Γ point, and solved straightforwardly.

Next, we solve the acoustic wave equation [5]

$$-\rho \partial_t^2 u_i + \partial_j (C_{ijkl} \partial_k) u_l = 0, \quad (4)$$

in the quasi-static regime, with the boundary conditions

$$u_j |_{\partial W_{\pm x}} = -D x \delta_{xj} |_{\partial W_{\pm x}}, \quad u_j n_j |_{\partial W \setminus \{\partial W_{\pm x}\}} = 0, \quad (5)$$

which gives the geometry of the unit cell under an $s_{\text{xx}}^{\text{eff}} = -D$ strain. Here C_{ijkl} denotes the stiffness tensor, u_i is the displacement, n_j is the normal vector at the boundary ∂W of the unit cell and D is a small parameter controlling the extent of the compression. The solution to this problem ultimately gives a compressed cell geometry with an interior strain field which alters the constituent permittivities. For this compressed geometry (with strained permittivities), we determine an effective permittivity using (3) once more to recover $p_{\text{xxxy}}^{\text{eff}}$, following from the analogue to (2). Next we consider the effective Brillouin line width of a cubic metamaterial.

IV. EFFECTIVE BRILLOUIN FREQUENCY SHIFT AND BRILLOUIN LINE WIDTH

To incorporate the leading-order effects of mechanical loss in our model, we incorporate the phonon viscosity tensor η_{ijkl} in (4). This is achieved by replacing the stiffness tensor C_{ijkl} with $C_{ijkl} + \eta_{ijkl} \partial_t$ [5], which subsequently perturbs acoustic frequencies as

$$\tilde{\Omega}^2 \rightarrow \tilde{\Omega}^2 + i \tilde{\Omega} \langle a_j \tilde{u}_j^* \rangle (\langle \rho \tilde{u}_j \tilde{u}_j^* \rangle)^{-1}, \quad (6)$$

where $a_i = \partial_j (\eta_{ijkl} \partial_k \tilde{u}_l)$ represents the overlap between the strain and the viscosity tensor. Evaluating the square root numerically, effective parameters are immediately obtained since $\tilde{\Omega} \rightarrow \tilde{\Omega}_B^{\text{eff}} - i \Gamma_B^{\text{eff}}/2$. This approach is valid provided that $\tilde{\mathbf{q}} = 2\mathbf{k}_1 = (0, 0, 4\pi n_{\text{eff}}/\lambda_1)$, corresponding to the SBS resonance, for the frequency $\tilde{\Omega}$ with corresponding longitudinal mode \tilde{u}_i . The effective acoustic velocity is given by $V_A^{\text{eff}} = \tilde{\Omega}/\tilde{q}$.

V. EXAMPLES

Using the methods outlined above for determining effective parameters, we consider a cubic lattice of As_2S_3 spheres in a silicon background material at $\lambda_1 = 1.55 \mu\text{m}$. The period of the cubic lattice is $d = 50 \text{ nm}$, to ensure the metamaterial is suitably subwavelength (i.e., there are approximately ten fundamental cells per optical or acoustic wavelength in the material). In Figure 1 we compute the maximum effective gain coefficient (that is, at $\Omega = \Omega_B$) as a function of filling fraction $f = 4\pi a^3/(3d^3)$ from $0 < f < 0.5$ (terminating close to the sphere touching limit at $f = \pi/6$). Figure 1a shows that the gain coefficient increases monotonically from the intrinsic value for pure silicon of $\max(g_P^{\text{eff}}) = 2.4 \times 10^{-12} \text{ m} \cdot \text{W}^{-1}$ to $\max(g_P^{\text{eff}}) = 1.06 \times 10^{-10} \text{ m} \cdot \text{W}^{-1}$ at $f = 50\%$, which is an enhancement factor of more than 40, and a gain coefficient now comparable to fused

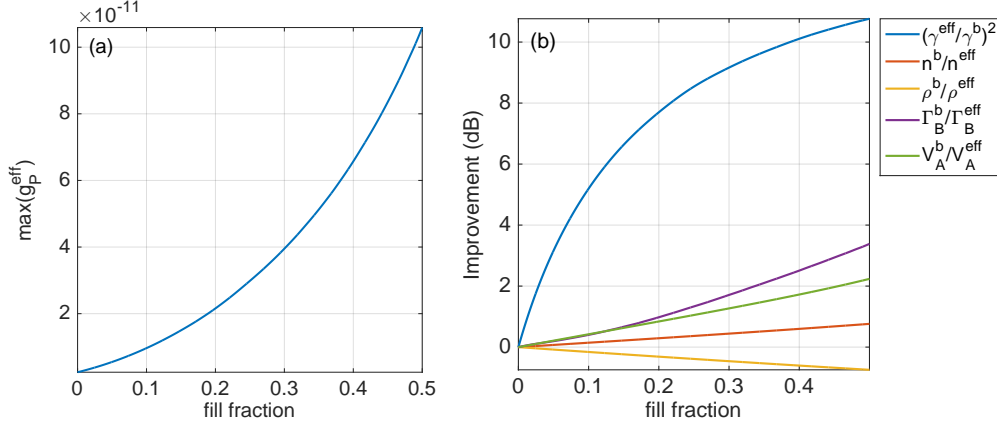


Fig. 1: Plots of (a) effective maximum gain coefficient, and (b) contribution from each term, as functions of filling fraction, for cubic lattice of As_2S_3 spheres in Si (where superscript b denotes background value)

silica [6]. To understand how each parameter contributes to the gain of the resulting metamaterial, we also evaluate $10 \log_{10}(g_P^{\text{eff}}/g_P^b)$. Using the logarithmic product rule, and arranging each term to ensure the total enhancement in the gain is given by simple addition of all parameter curve values at fixed f , we obtain the figure presented in Figure 1b. Note that the effective gain is driven largely by an increase in the effective electrostriction, and is more significant than reductions in the Brillouin linewidth, acoustic velocity, and refractive index combined. In this instance, the increase in the effective density works against the gain coefficient, however its effect is weak relative to the other terms.

VI. CONCLUSION

We have presented a robust method for determining the effective gain coefficient of a cubic metamaterial, outlining procedures for calculating the effective photoelastic parameter $p_{\text{xxyy}}^{\text{eff}}$, as well as the effective Brillouin line width Γ_B^{eff} for the first time. By introducing a cubic suspension of chalcogenide spheres, an order of magnitude enhancement in the bulk gain coefficient of silicon is achieved, which is extremely promising for designers of on-chip devices.

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