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Tunable electrostriction in metamaterials

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Abstract – **We characterise the optical, mechanical and acoustic properties of a medium through a theoretical examination of the electrostriction constant of both conventional dielectric materials, metals, and composites. The objective is to investigate whether one can construct metamaterials which exhibit tuneable electromechanical properties, for the purpose of engineering enhanced stimulated Brillouin scattering (SBS).**

I. Introduction

Electrostriction is the induced strain that arises in a material in response to an applied electric field [1]. This strain generates a pressure field which in turn gives rise to a propagating acoustic wave inside a medium. This acoustic wave results in a periodic variation in the optical properties of the material, essentially acting as a travelling diffraction grating. Typically an acoustic response is an undesired effect in optical systems, but it does in fact have direct practical applications, such as in the fabrication of fibre Brillouin lasers [2] and in the design of nanophotonic devices [3]. Accordingly, there is considerable interest in the fabrication of materials which exhibit strong electrostriction effects. To this end, we investigate whether composites can exhibit strong or weak electrostriction, and demonstrate that one can tune the SBS response of a material. The bases for this theoretical study are the analytical models of effective index theory [4, 5], which are then applied to the existing theory of electrostriction. Assuming suitable interface conditions, we obtain a closed form representation for the electrostriction of a composite for the first time.

II. Definition of electrostriction for conventional dielectric materials

The electromagnetic energy density u for an electrostatic problem with nondispersive and lossless material parameters [6] can be expressed in the form

$$u = \frac{1}{2}\epsilon_0\epsilon_r|\mathbf{E}|^2, \tag{1}$$

where ϵ_0 denotes the permittivity of free space, ϵ_r the permittivity of the dielectric and $\mathbf{E}$ the electric field vector. If one then differentiates (1) with respect to the density ρ and equates this change in the energy density with the work done by the system (assuming an adiabatic process) one obtains the pressure field contribution

$$P = -\frac{1}{2}\epsilon_0\rho\frac{\partial\epsilon_r}{\partial\rho}|\mathbf{E}|^2 = -\frac{1}{2}\epsilon_0\gamma|\mathbf{E}|^2, \tag{2}$$

where γ is defined as our electrostriction constant, and P is the strictive pressure [1]. Next we consider simple microscopic theories in order to obtain expressions for the electrostriction constants of conventional materials based on their constituent properties, for the purposes of investigating composite media.

III. Effective medium description of electrostriction for composites

In the long wavelength limit, we can obtain an effective permittivity of a dielectric ϵ_r^{d} from the polarisability of individual molecules in a material using the Clausius–Mossotti equation [4], which assumes that the medium is constructed as a three-dimensional dilute cubic or random lattice of molecules. One can then differentiate this expression and relate the resulting expression to the elasto-optic coefficients p_{ij} [7] of the medium. This permits an approximation of the electrostriction constant for the purposes of experiment in the form [8]

$$\gamma^{\mathrm{d}} = \frac{1}{3}(\epsilon_r^{\mathrm{d}})^2\,(p_{11} + 2p_{12})\,. \tag{3}$$

For metals, it is not possible to measure these elasto-optic coefficients and instead we consider a Drude model for the permittivity [9], which, after incorporating dispersion in u [6] admits the expression

$$\gamma^{\mathrm{m}} = (\epsilon_r^{\mathrm{m}} - \epsilon_r^{\infty}) + \frac{2\omega^2 - \mathrm{i}\omega\omega_\tau}{\omega_p^2}(\epsilon_r^{\mathrm{m}} - \epsilon_r^{\infty})^2. \tag{4}$$

For a composite material, we use the Maxwell–Garnett model [5] for the effective permittivity, and impose the additional constraint that the pressure fields remain in equilibrium. This implies that the resulting perturbations to these fields satisfy

$$\Delta P^i\big|_{\partial\Omega} = \Delta P^e\big|_{\partial\Omega}, \tag{5}$$

where $\Delta P^{i,e}$ denotes the change in pressure on the interior and exterior sides of the inclusion boundary $\partial\Omega$. Subsequently the electrostriction constant for a composite is given by

$$\gamma^{\mathrm{c}} = \frac{\beta_i f}{\beta_{\mathrm{c}}}\left[\frac{(\epsilon_r^i + 2\epsilon_r^e) - (\epsilon_r^i - \epsilon_r^e)}{(\epsilon_r^i + 2\epsilon_r^e) - f(\epsilon_r^i - \epsilon_r^e)}\right]^2 \gamma_i + \frac{\beta_e(1-f)}{\beta_{\mathrm{c}}}\left[\frac{(\epsilon_r^i + 2\epsilon_r^e)^2 + 2f(\epsilon_r^i - \epsilon_r^e)^2}{[(\epsilon_r^i + 2\epsilon_r^e) - f(\epsilon_r^i - \epsilon_r^e)]^2}\right]\gamma_e$$
$$+ \frac{(\beta_e - \beta_i)f(1-f)(\epsilon_r^i - \epsilon_r^e)}{\beta_{\mathrm{c}}}\left[\frac{3\epsilon_r^e(\epsilon_r^i + 2\epsilon_r^e)}{[(\epsilon_r^i + 2\epsilon_r^e) - f(\epsilon_r^i - \epsilon_r^e)]^2}\right], \tag{6}$$

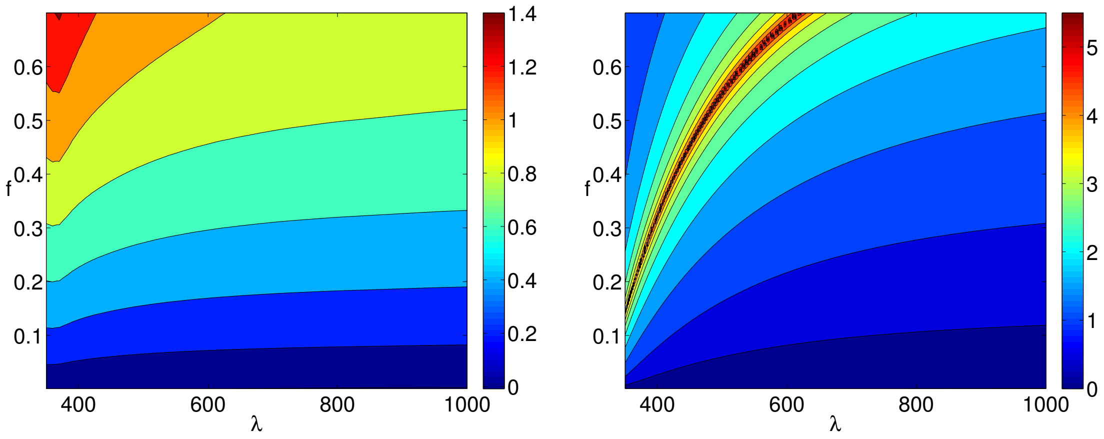
where $\epsilon_r^{\mathrm{i,e}}$, $\beta^{\mathrm{i,e}}$ and $\gamma^{\mathrm{i,e}}$ denote the permittivity, compressibility, and electrostriction constants of the interior and exterior domains within the unit cell of the cubic lattice, respectively. Here the compressibility is defined as $\beta = K^{-1}$, where K denotes the bulk modulus.

IV. Examples

As a simple demonstration of our model, we consider a cubic lattice of spheres embedded in a silica host and compute the composite electrostriction constant γ^{c}. To begin, we consider silicon spheres embedded in silica and observe in Fig. 1a that there is a linear enhancement in the electrostriction with filling fraction for almost all wavelengths, with the exception of a region near $\lambda \sim 370$ nm which corresponds to a short interval of zero dispersion for Si [10]. We note that the largely linear response observed in Fig. 1a for the Si-SiO_2 configuration is particularly interesting considering that $\gamma \sim 1$ for SiO_2 [8] and $\gamma < 0$ for Si [11]. In Fig. 1b we consider a metal-dielectric composite (Ag spheres in silica), where the behaviour is clearly nonlinear and the electrostriction extends over a much wider range. In this contour plot we observe a Maxwell–Garnett resonance arc, which corresponds to the effective permittivity in (6) becoming singular – a consequence of the filling fraction approaching the value $(\epsilon_r^i + 2\epsilon_r^e)/(\epsilon_r^i - \epsilon_r^e)$. This manifests as an extremely large increase in the electrostriction in the order of $\gamma^c \sim 10^5$ for this particular numerical discretisation of our parameter space. Physically we know that this arc explains why one observes red coloration in glass containing gold spheres [5], however for the purposes of investigating the electrostriction, we emphasise that suitable care must be exercised in the region of this resonance. This is because such regions corresponds to not only unbounded electrostriction values but also extremely high attenuation. Nevertheless, provided one is suitably far away from this peak, there are still regimes where both reasonable electrostriction enhancement and low attenuation can be achieved. This permits a practically realisable tuning of the electrostriction constant, which is fundamentally connected to SBS processes, to values which could be demonstrated experimentally.

V. Conclusion

We have extended effective index theory to the study of electrostriction and theoretically shown strong enhancement in the mechanical response of a material for dilute suspensions of metallic spheres in silica, as well as reasonable enhancement for dielectric spheres embedded in a dielectric host.



(a) (b)

Fig. 1: Contour plots of the composite electrostriction $\log_{10}|\mathrm{Re}(\gamma^c)|$ for (a) Si spheres-in-SiO_2, and (b) Ag spheres-in-SiO_2 as a function of wavelength λ (nm) and filling fraction f.

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